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CONTROL AND STABILITY STUDIES OF BALL AND BEAM SYSTEM

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Abstract. The commonest benchmark for testing control algorithms in control engineering laboratories is the ball and beam system. It is vastly unstable as well as nonlinear. An example of its application is found in balancing the ball on the beam, which is analogous to stabilizing an airplane horizontally during landing and or in turbulent air-flow. In this paper, pole placement (PP) and proportional integral derivative (PID) controllers were designed to control the ball position on the beam. Comparative analysis was presented. The performance indices used were the Integral absolute error, time response analysis, and integral square error. Based on the analysis and simulation results, better performances were recorded with PP.

Keywords: Balancing, stability analysis, benchmark, airplane, integral square error.
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1 Introduction

The ball and beam system, because of its significance in control applications, is one of the most popular laboratory design experiment. It is a common feedback control system due mostly to its ease of construction and its use in learning. The basic components of this system is a motor, attached to the beam such that the shaft of the motor can help to manipulate the positions of the ball on the beam by controlling the angle of tilt of the beam. This is done with the use of a lever arm attached to the end of the beam. These types of systems have wide areas of applications in industries including passengers' platform balancing for comfort, control of rocket and aircraft take-off and landing. This system is an open-loop system which is unstable since for a given a constant angle of tilt on the beam, the ball position changes without limit. Due to the instability of the system and non-linearity property in the system's model, there is, therefore, need for some forms of a feedback loop to control the ball's positions and to design a robust controller for the system's model so as to have the best performance in real-time applications.

Many researchers have taken this upon themselves to design different controllers to improve the performance of the system. Taifour et al. (2017) designed a PID controller for a ball and beam system to control the ball position. These controllers are designed based on two feedback loops: the inner and the outer loops. The inner loop controls the motor gear angle position while the outer loop uses the inner loop feedback to control the ball position. Keshmiri et al.

(2012) was based on designing two control strategies and comparing their performances on the system, taking into consideration the coupling effect on the dynamics equation of the system. Therefore, the authors designed a PID, model-based controller and LQR and non- model-based controllers. The parameters of the LQR are tuned using GA. The authors also designed a state observer to monitor the velocity of the ball due to the noise on the system. The result of the work showed that the model-based outperforms the non-model based. Rahmat et al. (2010) focused on designing conventional, modern and intelligent controllers and analyze the performance of these controllers on the ball and beam system. The result of the design revealed that the PID controller showed better performance among others and that the intelligent controller can be improved by using advanced configuration and better tuning. Salem (2013) looked into the conceptualization and development of ball and beam system based on mechatronics designed approach. The design of complete and subsystem, simulation and analysis was presented. The author developed a PD controller for the system and simulation results were verified using MATLAB Simulink and Proteus software. The authors in Colon et al. (2013), after linearizing the system, applied the pole placement technique in order to control the ball and beam system. A second controller is then designed using LQR/LTR, where the robustness of the system was taken into consideration and this controller was able to reduce oscillation and performed better than the first. The conclusion from the work showed that non-linear controllers are more effective than linear controllers. In Shirke & Kulkarni (2015), two controllers were designed to control and test the system responses. The first controller has modified PD controller and the second controller was modified PD whose parameters were optimized using PSO techniques was designed. The controller was called PD-PSO controller. The author then compared the system's responses with PD with modified PD and modified PD-PSO controller. The result showed that the PD-PSO controller presents satisfactory performances and pose good robustness. In their work Rosales (2004) built the ball and beam system which is made of acrylic with the first configuration. The linear potentiometer technique was designed for monitoring the ball position on the beam and the encoder sensor to measure the angle of the beam. In this model, the ball position is controlled by an inner motor loop and an outer ball position loop using d SPACE controller board. The experimental result revealed that the ball with the small step command tracked the position quickly without overshoot. However, the ball still tends to overshoot and oscillate several times before coming to rest. In Hung et al. (2017), control strategies based on the optimal control synthesis which include LQR and H_2 optimization to manipulate the ball and beam system control was designed. The sensing of the ball is done using distance sensors and the design used a DSPTMS320F2883 data acquisition card to implement the control algorithm. The researchers in Reza & Minh (2016); Moezzi et al. (2018); Amjad et al. (2010) designed PID and fuzzy logic to control the ball and beam system. In Reza & Minh (2016); Moezzi et al. (2018) PID controller and two fuzzy logic controllers based on Mamdami and Sugeno inference systems to control ball and beam system were designed. Also, in this paper, the response of the system to different references such as the step, sinusoidal and square waves was investigated. The result of the design showed that fuzzy logic is superior over PID in tracking square wave. While in Amjad et al. (2010), the authors designed three difference PID controllers and compared their performances to the fuzzy logic controller. The work in Muawia et al. (2014) is based on designing a fuzzy static and fuzzy dynamic to control the motion of a ball and beam system. The result of this investigation showed that fuzzy static can control the ball moves faster than that of the fuzzy dynamic. Saad & Khalfallah (2017) also designed a PID controller algorithm using Arduino microcontroller to control ball position using the potentiometer position sensor while Saad & Khalfallah (2019) proposed a new control strategy to control the position of the ball by adopting an active disturbance rejection control (ADRC). The literature Valluru (2016) addresses the control and real-time modeling of the ball and beam system. Performance analysis was carried out using PID, state space, lag-lead, robust LQR and observer-based on LQG controllers. The Lyapunov direct method was used to describe the unstable behavior of the ball and beam system where the authors showed that the robust LQR controller approaches the desirable performance when compared with the other controllers. Rana et al. (2011) used Particle Swamp Optimization algorithm to tune the gains of PID controllers to control ball and beam system (Burakov, 2017) developed a fuzzy-modal controller for the ball-beam position control while Ali & Kumar (2013) designed self- adaptive fuzzy PID controller and fuzzy controller in order to control ball and beam system. In this paper, pole placement and PID control schemes were proposed to stabilize the ball and beam system and track the ball position on the beam.

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2 Ball and Beam System

A ball is placed on a beam, as in Taifour et al. (2017) where it is allowed to roll along the length of the beam. The lever arm is attached to the beam at one end and a servo gear at the other. As the servo gear turns by an angle θ, the lever changes the angle of the beam by $\propto$. When the angle is changed from the horizontal position, the ball is caused to roll along the beam by gravity. A controller will be designed for this system so that the ball's position can be manipulated.

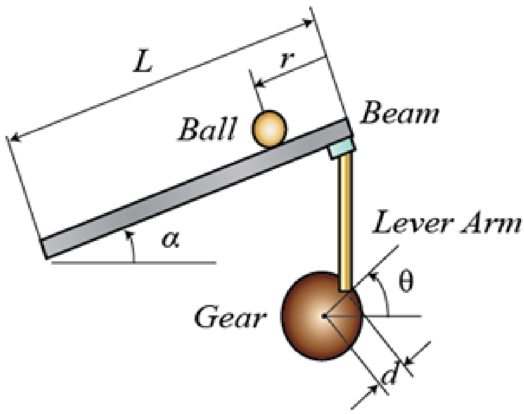

Figure 1: Ball and beam model (Taifour et al., 2017)

Table 1: System Parameters

Parameter	Value /Unit
Mass of the ball (m)	0.11 kg
Radius of the ball (R)	0.015 m
Lever arm offset (d)	0.03 m
Acceleration due to gravity(g)	9.81ms^{-2}
Length of the beam (L)	1.0 m
Ball's moment of inertia (J)	$9.99e-6kg.m^2$

3 Ball and Beam Dynamics

The nonlinear equation of motion for the ball using the Lagrangian function as in Taifour et al. (2017) is:

$$0 = \left(\frac{J}{R^2} + m\right)\ddot{r} + mgsin \propto - mr\dot{\propto}^2. \tag{1}$$

Linearizing equation (1) about the beam angle α =0, yields:

$$\left(\frac{J}{R^2}+m\right)\ddot{r}=-mg\propto. \tag{2}$$

The approximated linear equation that relates the beam angle to the angle of the gear can be written as:

$$\propto=\frac{d}{L}\theta. \tag{3}$$

Using this to replace α in equation (C) yields:

$$\left(\frac{J}{R^2}+m\right)\ddot{r}=-mg\frac{d}{L}\theta. \tag{4}$$

Laplace transforms of equation (D) is;

$$\left(\frac{J}{R^2}+m\right)R\left(s\right)s^2=-mg\frac{d}{L}\theta(s). \tag{5}$$

Hence the transfer function is as in [1];

$$P\left(s\right)=\frac{R(s)}{\theta(s)}=-\frac{mgd}{L(\frac{J}{R^2}+m)}\frac{1}{s^2}\qquad[\frac{m}{rad}]. \tag{6}$$

Selecting the ball's position (r) and velocity ($\dot{r}$) as the state variables and gear angle (θ) as the input, the state space representation is as in Taifour et al. (2017);

$$\begin{bmatrix}\dot{r}\\ \ddot{r}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}r\\ \dot{r}\end{bmatrix}+\begin{bmatrix}0\\ -\frac{mgd}{L(\frac{J}{R^2}+m)}\end{bmatrix}\theta. \tag{7}$$

4 Controller Design

In this section, PP and PID controllers were designed for tracking control of ball position and comparative studies of these control schemes were also conducted. The control measures which can be used to compare the quality of the controlled responses of different control schemes or different sets of tuning parameters are the Integral Absolute Error (IAE) and the Integral Square Error (ISE). Thus, this is performance indices employed in this work as proposed by Hussain et al. (2014) to take care of the error and compare the performances of the controllers.

4.1 Pole Placement

In this section, a model-based pole placement control scheme was proposed. Design specification and state-space model of the system was used in obtaining the controller gain. Figure 2 shows the block diagram of the control scheme, where K is the controller gain and N is the reference input to take care of the steady-state error. In this work, the design criteria are given as; Overshoot of 5% and settling time of 3 seconds.

The poles of the system, N and K values were obtained using MATLAB command as;

$$K=[0.0934\ 75.1694-38.8872\ 0.0195]\ \ and\ \ N=0.0934.$$

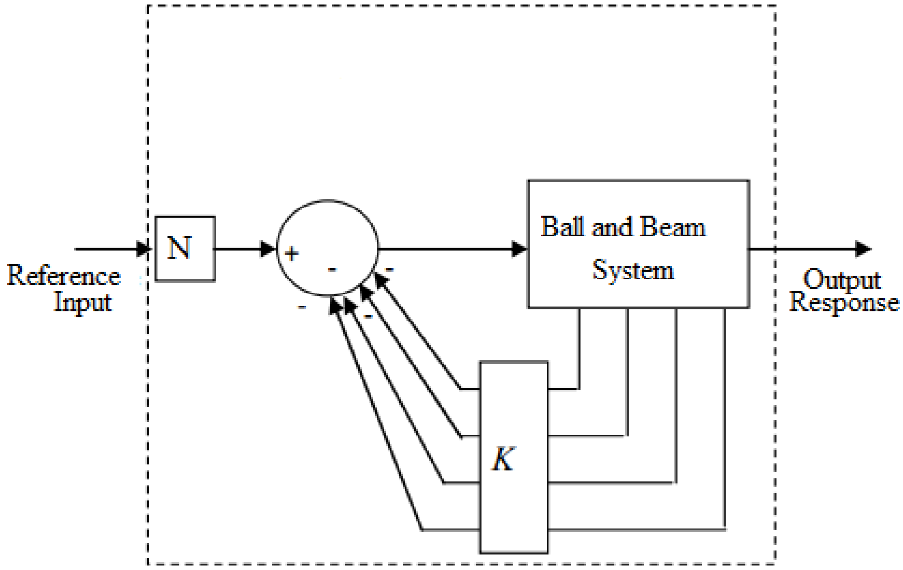

Figure 2: Block diagram of the control scheme

4.2 PID Controller

PID Controller stands for Proportional Integral Derivative Controller. The concept of PID is explained in this section. Combining the three control actions of PID, the general expression is obtained as in equation (8);

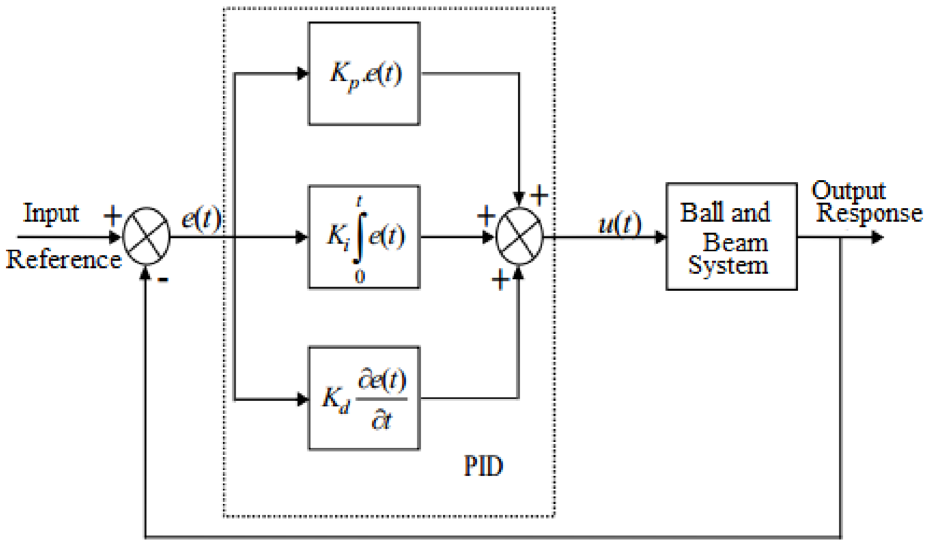

Figure 3: Block diagram of the control scheme

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt} \tag{8}$$

Where $u(t)$ is the control signal, K_d, K_i, K_p are derivative, integral and proportional gains respectively and $e(t)$ is the error due to the difference between the response signal and the desired signal. In tuning PID to achieve desired response; proportional gain enhances response speed, however, faster response causes a steady-state error. A good solution for the steady-state error is the integral gain. Overshoot is another undesired behavior of the response; it can be fixed by derivative gain. The gains were obtained as; $K_d = 38,\ K_i = 0.07,\ K_p = 14.8$. Figure 3 is the block diagram representation of PID control scheme for a ball and beam system where the output of PID controller is applied to the ball and beam system and the output of the system is feed backed to be compared with the reference input. Tuning is done to achieve negligible error.

5 Results and Discussions

In this work, two dissimilar control strategies were designed and applied on ball and beam system for stability and ball position tracking control. The design is based on the following design criteria; Overshoot of 5% and Settling time of 3 seconds which is achieved when the response is less the 2% of its final value. The system is type II which means that it has double poles at the origin, thus the open-loop system is highly unstable and nonlinear. When the voltage is applied to the open-loop system, the ball will roll right off the beam end. Figure 4 shows the bode plot of the open-loop system with a phase margin of zero, hence unstable, the purpose of the bode plot is the estimation of the closed-loop response via adding a controller to the system. While figure 5 shows the closed-loop bode plot with a phase margin of *85*, using the PID controller, the system was stabilized. The two control schemes of PP and PID were designed for ball position tracking control as showed in figures 6 and 7, the time response specification which is rise time, settling time, overshoot and steady-state error obtained with PID control action are 0.28s, 2.13s, 2.86%, and 2.2 respectively. However, with PP control actions better response specifications of 0.36s, 0.86s, 0.01%, and 1.87 respectively was achieved. This is as recorded in Table 2. Based on the results obtained, the design criteria were satisfied. Modern control systems are often complex and require more sophisticated performance criteria and so, error and time are the important factors that must be considered. IAE and ISE are two performance indexes that were considered in this work, thus the 3.94×10^{-4} and 4.11×10^{-4} were recorded for IAE and ISE respectively using PP control action while 4.76×10^{-4} and 5.24×10^{-4} were recorded for the same performance indexes but using the PID control schemes. Figure 8, and Table 3 compared the two control strategies and assessed their performances.

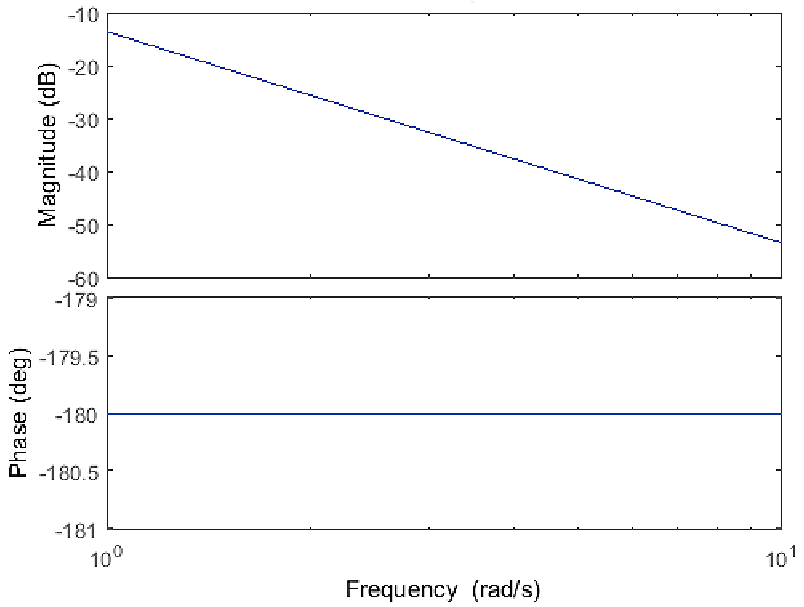

Figure 4: Open-loop bode plot

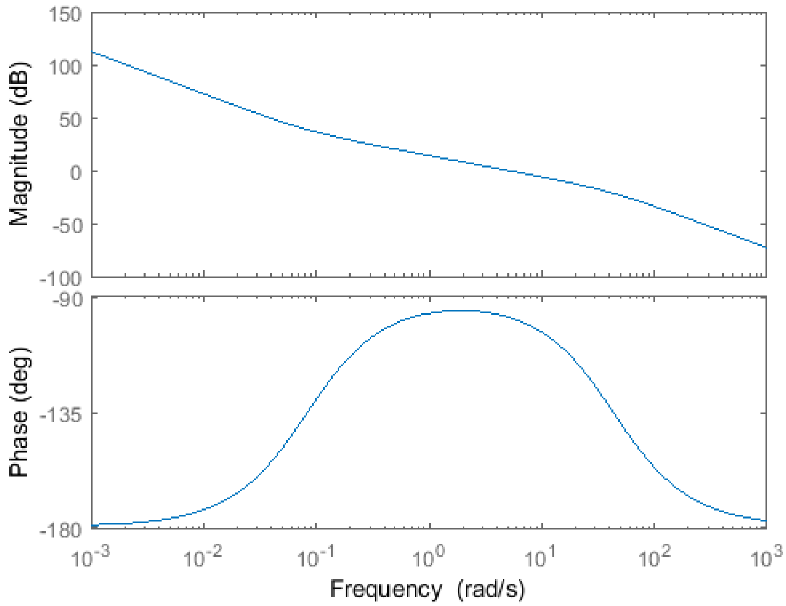

Figure 5: Closed-loop bode plot

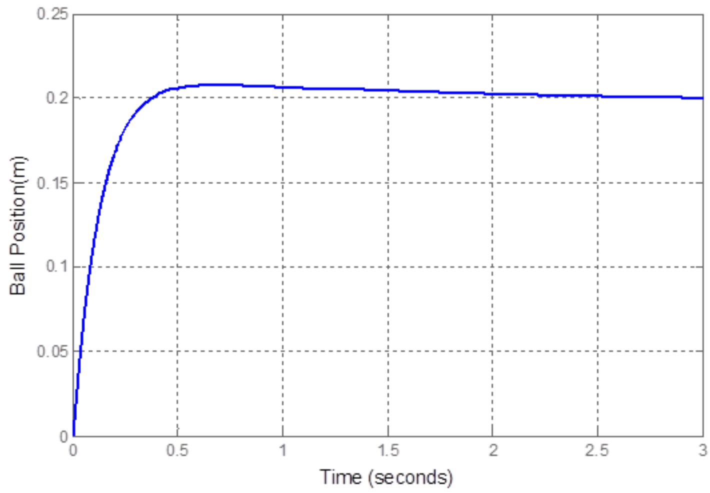

Figure 6: PID controller for ball position tracking

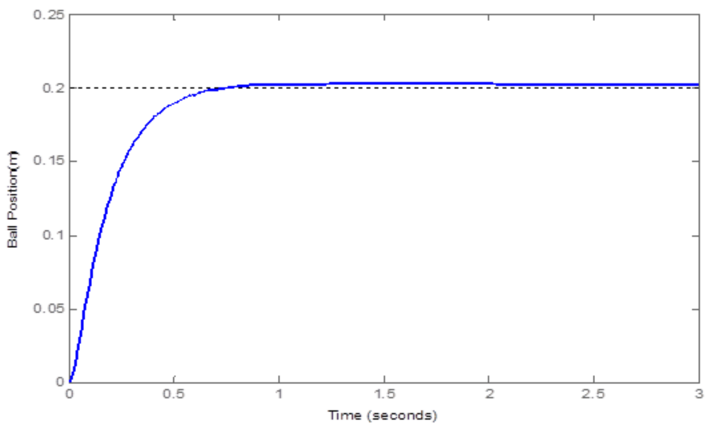

Figure 7: Pole placement controller for ball position tracking

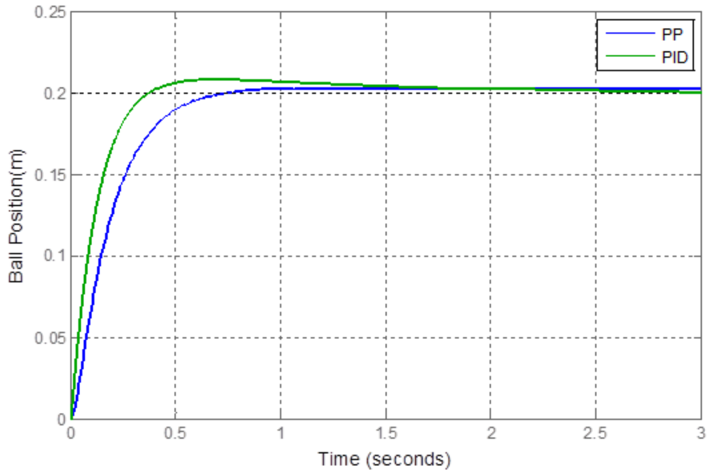

Figure 8: Comparing the performance of the two control scheme

Table 2: Time respond specifications

Controllers	Rise Time (sec)	settling time (sec)	Overshoot (%)	Steady-state error (%)
PP	0.36	0.86	0.01	1.87
PID	0.28	2.13	2.86	2.2

Table 3: Performance indexes

Controllers	ISE	UAE
PP	4.11x10^{-4}	3.94x10^{-4}
PID	5.24x10^{-4}	4.76x10^{-4}

6 Conclusions

In this work, a mathematical model of ball and beam system was presented where PID and PP were designed based on the design specification to stabilize and control the system. PID was designed using the manual tuning method while PP was designed using MATLAB command. Stabilization and ball position tracking control was achieved using the two control schemes. A comparative assessment was achieved via the use of time response, integral square error and integral absolute error as the performances indices for the system. Both controllers meet the design specifications. However, pp shows superior performances to PID.

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